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Thirring Model in Lower Dimensions: Nonperturbative Approaches[a]

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A reformulation of the Thirring model as a gauge theory on both continuum space-time and discretized lattice is reviewed. In (1+1) dimensions, our result reproduces consistently the bosonization of the massless Thirring model. In (2+1) dimensions, the analysis by use of Schwinger-Dyson equation is shown to exhibit dynamical fermion mass generation when the number N of four-component fermions is less than the critical value $N_{\rm cr} = 128/3\pi^2$.

1 Gauge Theory Formulation of Thirring Model

The Lagrange density of Thirring model is given by

$$\mathcal{L}_{\rm Thi} = \sum_a \bar{\psi}_a i\gamma^\mu \partial_\mu \psi_a - \frac{G}{2N} \sum_{a,b} \bar{\psi}_a \gamma^\mu \psi_a \, \bar{\psi}_b \gamma_\mu \psi_b. \tag{1}$$

Here ψ_a is treated as a formal one in arbitrary dimensions, but a four-component Dirac spinor in (2+1) dimensions and a, b are summed over from 1 to N. (For the notations and the detailed calculations, see Ref.[1,2].) Since the form of four-fermion interaction term is a contact term between vector currents,[b] a well-known technique to facilitate $1/N$-expansion is to rewrite Eq.(1) by introducing an auxiliary vector field A^μ such as

$$\mathcal{L}_{\rm aux} = \sum_a \bar{\psi}_a i\gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi_a + \frac{1}{2G} A_\mu A^\mu. \tag{2}$$

Gauge-noninvariant as the above Lagrangian is, however one may be tempted to regard A_μ as a gauge field. A systematic way to construct the $U(1)$ gauge theory, but is gauge equivalent to Eq.(2) is to elicit the fictitious Goldstone

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[b] When we consider the model of many fermion flavors, another current-current type four fermion contact term $\sum_{a,b} \bar{\psi}_a \gamma^\mu \psi_b \, \bar{\psi}_b \gamma_\mu \psi_a$ is allowed.

degree (or equivalently the Stückelberg field) based on the principle of hidden local symmetry [3,4,1,5]:

$$\mathcal{L}_{\rm HLS} = \sum_a \bar{\psi}_a i\gamma^\mu D_\mu \psi_a + \frac{1}{2G}(A_\mu - \sqrt{N}\partial_\mu \phi)^2. \tag{3}$$

It is obvious that Eq.(3) possesses a $U(1)$ gauge symmetry and the gauge-fixed (unitary gauge) form of it exactly coincides with Eq.(2), so does the original Thirring model in Eq.(1). The gauge theory formulation of Thirring model in Eq.(3) looks like quantum electrodynamics (QED) + scalar QED with kinetic term of the gauge field and the Higgs degrees of freedom truncated in the tree level. Now that we find a gauge-invariant formulation of the Thirring model, we have the privilege to choose the gauge appropriate for our particular purpose. Here, instead of the unitary gauge notorious for loop calculations, let us consider the nonlocal R_ξ gauge at the Lagrangian level

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}\Big(\partial_\mu A^\mu + \sqrt{N}\frac{\xi(\partial^2)}{G}\phi\Big)\frac{1}{\xi(\partial^2)}\Big(\partial_\nu A^\nu + \sqrt{N}\frac{\xi(\partial^2)}{G}\phi\Big), \tag{4}$$

where the gauge fixing parameter ξ has the momentum- (derivative-) dependence. It is straightforward to prove the possession of the BRS symmetry in spite of the nonlocality of ξ and thereby it guarantees the S-matrix unitarity. Another intriguing point is that, in the combined Lagrangian of Eqs. (3) and (4), the fictitious Nambu-Goldstone boson ϕ is completely decoupled independently of the specific form of $\xi(\partial^2)$. Though we introduced $U(1)$ gauge symmetry by use of the flavor-singlet current, the hidden local symmetry can easily be extended to the non-Abelian one, *e.g.* $U(n)_{\rm global} \times U(n)_{\rm local}$ symmetry by use of the "$U(n)/U(n)$" nonlinear sigma model :

$$\mathcal{L}_{Non-Ab.} = \sum_a \bar{\psi}_a i\gamma^\mu D_\mu \psi_a - \frac{N}{G} tr\left[(D_\mu u \cdot u^\dagger)^2\right], \tag{5}$$

where $A_\mu = A_\mu^\alpha T^\alpha$, and $u = e^{i\phi}, \phi = \phi^\alpha T^\alpha$, with T^α being the $U(n)$ generators. Actually, Eq.(5) is gauge equivalent to the Thirring model having the interaction

$$-\frac{G}{2N}\sum_{a,b,\alpha}(\bar{\Psi}_a \gamma^\mu T^\alpha \Psi_a)(\bar{\Psi}_b \gamma_\mu T^\alpha \Psi_b). \tag{6}$$

In contrast to the $U(1)$ case, however, the fictitious NG bosons ϕ in the non-Abelian case are not decoupled even in the R_ξ gauge, which would make the analysis at quantum level rather complicated.

Now that we have reformulated the Thirring model as a gauge theory, we can further gain an insight into the theory by using a technique inherent to the gauge theory, namely, the dual transformation [6]. We first consider the path integral for the Lagrangian (3), and linearize the "mass term" of gauge field by introducing an auxiliary field C_μ. Through an integration over the scalar field ϕ,[c] we obtain a delta functional for $\partial_\mu C^\mu$. If we pick up C_μ by use of dual antisymmetric-tensor field $H_{\mu_1\cdots\mu_{D-2}}$ of rank $D-2$, which satisfies the Bianchi identity, we obtain the following path integral after integrating out the auxiliary field C_μ:

$$
\begin{aligned}
Z_{\rm Dual} &= \int [dH_{\mu_1\cdots\mu_{D-2}}][dA_\mu][d\bar{\psi}_a][d\psi_a] \exp i \int d^D x \Big\{ \sum_a \bar{\psi}_a i\gamma^\mu D_\mu \psi_a \\
&\quad + \frac{(-1)^D}{2(D-1)} H_{\mu_1\cdots\mu_{D-1}} H^{\mu_1\cdots\mu_{D-1}} + \frac{1}{\sqrt{G}} \epsilon^{\mu_1\cdots\mu_D} A_{\mu_1} \partial_{\mu_2} H_{\mu_3\cdots\mu_D} \Big\} \qquad (7) \\
&= \int [dH_{\mu_1\cdots\mu_{D-2}}][d\bar{\psi}_a][d\psi_a]\, \delta\Big(\sum_a \frac{1}{\sqrt{N}} \bar{\psi}_a \gamma^{\mu_1} \psi_a + \frac{1}{\sqrt{G}} \epsilon^{\mu_1\cdots\mu_D} H_{\mu_2\cdots\mu_D}\Big) \\
&\quad \exp i \int d^D x \Big\{ \sum_a \bar{\psi}_a i\gamma^\mu \partial_\mu \psi_a + \frac{(-1)^D}{2(D-1)} H_{\mu_1\cdots\mu_{D-1}} H^{\mu_1\cdots\mu_{D-1}} \Big\}, \qquad (8)
\end{aligned}
$$

where $H_{\mu_1\cdots\mu_{D-1}} = \partial_{\mu_1} H_{\mu_2\cdots\mu_{D-1}} - \partial_{\mu_2} H_{\mu_1\mu_3\cdots\mu_{D-1}} + \cdots + (-1)^D \partial_{\mu_{D-1}} H_{\mu_1\cdots\mu_{D-2}}$. The Lagrangian (8) describes N "free" fermions and a "free" antisymmetric tensor field of rank $D-2$ which are, however, constrained through the delta functional. This implies that the dual field is actually a composite of the fermions.

Let us write down the discretized Lagrange density for the Thirring model on the 3D Euclidean lattice. Of course, it looks not so good way to use the original Lagrange density with four-fermion contact term in Eq.(1) directly. Therefore, one way is to discretize Eq.(2): [7]

$$
\mathcal{L}_{DH} = \sum_a \Phi_a^\dagger (\tilde{M}^\dagger \tilde{M})^{-1} \Phi_a - \frac{N}{2} \beta \sum_\mu \tilde{\theta}_\mu^2(x), \qquad (9)
$$

where Φ is the pseudo fermion, $\tilde{\theta}_\mu$ is the vector auxiliary field, $\beta = 1/G\Lambda$ is the inverse of the dimensionless coupling rescaled by the ultraviolet cutoff Λ

[c] The scalar phase ϕ can in fact be divided into two parts: $\phi = \Theta + \eta$, where Θ expressed by multi-valued function describes the topologically nontrivial sector, e.g., the creation and annihilation of topological solitons, and η given by single-valued function depicts the fluctuation around a given topological sector. Inclusion of the topological sector Θ induces a topological interaction term [6], though we neglect Θ contribution in this section, since we are interested in ϕ as the Nambu-Goldstone mode.

which is identified with the inverse of the lattice size a, $1/\Lambda = a$, and

$$M_{x,y} = \frac{1}{2}\sum_{\mu}\eta_\mu(x)[(1+i\tilde{\theta}_\mu(x))\delta_{y,x+\mu} - (1-i\tilde{\theta}_\mu(x-\mu))\delta_{y,x-\mu}] + m\delta_{x,y}. \quad (10)$$

In Eq.(10), $\eta_\mu(x)$ is staggered fermion phase factor, and m a finite bare fermion mass term. The other method is to adopt the gauge theory formulation and take advantage of various benefits coming from the gauge theory formulation: [2]

$$\mathcal{L}_L = \sum_a \Phi^\dagger{}_a(M^\dagger M)^{-1}\Phi_a - N\beta\sum_\mu \cos\left(\phi(x+\mu) - \phi(x) + \theta_\mu(x)\right), \quad (11)$$

where Φ is the pseudo fermion and θ_μ is the gauge field,

$$M_{x,y} = \frac{1}{2}\sum_\mu \eta_\mu(x)(e^{i\theta_\mu(x)}\delta_{y,x+\mu} - e^{-i\theta_\mu(x-\mu)}\delta_{y,x-\mu}) + m\delta_{x,y}. \quad (12)$$

Here $\theta_\mu(x)$ is a gauge field which appears in the Lagrangian in terms of the link variable $\exp(i\theta_\mu)$. Note that the second term in Eq.(11) is so-called the annealed XY model with the gauge connection between the nearest neighbors in statistical mechanics.

2 Bosonization of the Thirring Model

In (1+1) dimensions the relation in the delta functional in Eq.(8) implies nothing but the bosonization condition in the scheme of path integral, i.e., $\frac{1}{\sqrt{G}}\epsilon^{\mu\nu}\partial_\nu H \approx \frac{-1}{\sqrt{N}}\bar{\psi}_a\gamma^\mu\psi_a$. Integrating the fermions in Eq.(7), we obtain an effective theory which consists of a pseudoscalar and a vector gauge fields:

$$Z_{2D} = \int [dH][dA_\mu]\,(\det i\not{D})^N \ \exp i\int d^2x \left\{\frac{1}{2}\partial^\mu H\partial_\mu H + \frac{1}{2\sqrt{G}}H\epsilon_{\mu\nu}F^{\mu\nu}\right\}. \quad (13)$$

The second term of the action in Eq.(13) is the (1+1) dimensional analogue of axion term which is the interaction term between the scalar and the gauge fields and takes the form $H\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ in (3+1) dimensions. Though the computation of fermionic determinant with regularization generates the Abelian chiral anomaly, this problem is resolved by the constant shift of scalar field H in axion term. Since the fermionic determinant is computed in an exact form, i.e.,

$$-iN\ln\frac{\det i\not{D}}{\det i\not{\partial}} = \frac{1}{2\pi}\int d^2x\ A^\mu(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2})A^\nu,$$

the integration over A_μ gives a free massless scalar theory as the bosonized Thirring model

$$Z_{\rm boson} = \int [dH] \exp i \int d^2x \, \frac{1}{2}(1+\frac{\pi}{G})\partial_\mu H \partial^\mu H. \tag{14}$$

If G is in the region $-\pi < G < 0$, the energy per unit volume is unbounded below and hence the (1+1)-dimensional Thirring model with coupling constant G ($G > 0$ or $G < -\pi$) is well-defined.

Though there have been several attempts to interpret the relation in Eq.(8) as that for the bosonization in arbitrary dimensions [8], their achievement is unclear yet since the fermionic determinant in spacetime dimensions more than (1+1)D produces nonlocal terms of the dual field.

3 Dynamical Symmetry Breaking

In (2+1) dimensions, our Lagrangian in Eq.(3) is invariant under the parity

$$\psi_a(x) \mapsto \psi_a^{'}(x^{'}) = i\gamma^3\gamma^1\psi_a(x), \quad A_\mu(x) \mapsto A_\mu^{'}(x^{'}) = (-1)^{\delta_{\mu 1}} A_\mu(x). \tag{15}$$

and the so-called global "chiral" transformation

$$\psi_a \mapsto \psi_a^{'} = \Big(\exp\big(i\omega^{i\alpha}\frac{\Sigma^i}{2} \otimes T^\alpha\big)\psi\Big)_a, \tag{16}$$

where $\Sigma^0 = I,\ \Sigma^1 = -i\gamma^3,\ \Sigma^2 = \gamma^5,\ \Sigma^3 = -\gamma^5\gamma^3$ and T^α denote the generators of $U(N)$. The question we shall address from now on is "which symmetry is broken dynamically?" In concern with the parity, first issue is whether one can take the regularization to keep both the $U(1)$ gauge symmetry and the parity or not. Since our gauge action in tree level has the parity-conserving mass and the number of two component Dirac fermion species is even, the parity need not be violated by appropriate regulator. For example, the introduction of parity-conserving Pauli-Villars regulator leads to the parity-invariant effective action for the gauge field as have done in (2+1)D quantum electrodynamics(QED_3) [9]. Another question is whether the pattern of dynamically-generated fermion mass involves the parity violating mass $(-m\bar{\psi}_a\gamma^5\gamma^3\psi_a)$ or not. Though at this stage we do not yet know whether the dynamical symmetry breaking really occurs or not, in this gauge-invariant formulation of the Thirring model, such symmetry breaking pattern is proven to be energetically unfavorable by using the exact argument in Ref. [10]. Namely, since the tree-level gauge action corresponding to Eq.(3) is real and positive semi-definite in Euclidean space, energetically favorable is a parity conserving configuration consisting of half

the 2-component fermions acquiring equal positive masses and the other half equal negative masses.

According to the above arguments, the pattern of symmetry breaking we shall consider is not the parity but the chiral symmetry, i.e., of which the breaking is $U(2N) \to U(N) \times U(N)$. Thus we investigate the dynamical mass of the type $m\bar{\psi}\psi$ in the Schwinger-Dyson equation, giving

$$\begin{aligned}
&(A(-p^2)-1)\not{p} - B(-p^2) = \\
&\quad -\frac{1}{N}\int \frac{d^D q}{i(2\pi)^D}\gamma_\mu \frac{A(-q^2)\not{q} + B(-q^2)}{A^2(-q^2)q^2 - B^2(-q^2)}\Gamma_\nu(p,q)\; iD^{\mu\nu}(p-q),
\end{aligned} \quad (17)$$

where the full fermion propagator is written as $S(p) = i[A(-p^2)\not{p} - B(-p^2)]^{-1}$, and $\Gamma_\nu(p,q)$ and $D_{\mu\nu}(p-q)$ denote the full vertex function and the full gauge boson propagator, respectively. Task is, by employing some appropriate approximations, to reduce Eq.(17) to the tractable integral equation for the mass function $M(-p^2) = B(-p^2)/A(-p^2)$. First, we here adopt the $1/N$ expansion for $\Gamma_\nu(p,q)$ and $D_{\mu\nu}(p-q)$ under a nonlocal R_ξ gauge, i.e., they are the bare vertex and the one-loop vacuum polarization of massless fermion loop at the $1/N$ leading order. Then the Schwinger-Dyson equation (17) becomes the coupled integral equations for $A(-p^2)$ and $B(-p^2)$. They support a trivial solution $A(-p^2) = 1$ and $B(-p^2) = 0$ at the $1/N$ leading order, however, as was realized in QED_3, we expect to find a nonperturbative nontrivial solution by examining them for finite N.

A way is, by use of the freedom of gauge choice, to require $A(-p^2) = 1$ in a Schwinger-Dyson equation for $A(-p^2)$. Then this gauge fulfills the consistency between the bare vertex approximation and the Ward-Takahashi identity for the hidden local $U(1)$ symmetry (or the current conservation), i.e., $A(0) = 1$. The specific form of the gauge is determined by a Schwinger-Dyson integral equation for $A(-p^2)$, and it reduces the coupled Schwinger-Dyson equations into a single equation for $B(-p^2)$ which turns out to be a mass function, i.e., $M(-p^2) = B(-p^2)$:

$$B(p^2) = \frac{1}{N}\int_0^{\Lambda^{D-2}} d(q^{D-2})K(p,q;G)\frac{q^2 B(q^2)}{q^2 + B^2(q^2)}, \quad (18)$$

where Λ is ultraviolet cutoff and the kernel $K(p,q;G)$ is given by

$$K(p,q;G) = \frac{1}{(D-2)2^{D-1}\pi^{(D+1)/2}\Gamma\left(\frac{D-1}{2}\right)}\int_0^\pi d\theta \sin^{D-2}\theta\; d(k^2)[D - \eta(k^2)], \quad (19)$$

with $k^2 = p^2 + q^2 - 2pq\cos\theta$. The gauge fixing parameter ξ is a function of k^2

such as

$$
\begin{aligned}
\eta(k^2) &= \frac{-\xi(k^2)C_D^{-1}k^{D-2}+k^2}{\xi(k^2)G^{-1}+k^2} && (20)\\
&= (D-2)\left[\left(1+\frac{Gk^{D-2}}{C_D}\right){}_2F_1\left(1,1+\tfrac{D}{D-2},2+\tfrac{D}{D-2};-\tfrac{Gk^{D-2}}{C_D}\right)-1\right], && (21)
\end{aligned}
$$

where $C_D^{-1} \equiv \frac{2\,tr\,I}{(4\pi)^{D/2}}\Gamma\left(2-\frac{D}{2}\right)B\left(\frac{D}{2},\ \frac{D}{2}\right)$ and ${}_2F_1(a,b,c;z)$ the hypergeometric function. Note that the kernel $K(p,q;G)$ is positive-definite for positive arguments p, q, G and is symmetric under the exchange of p and q.

Now let us show the existence of the nontrivial solution for the Schwinger-Dyson equation (18) when $2<D<4$. Since we have in mind the continuous phase transition, the solution of our interest is the nontrivial solution which starts to exist without gap in the vicinity of the phase transition point. Such a bifurcation point is identified by the existence of an infinitesimal solution $\delta B(p^2)$ around the trivial solution $B(p^2)=0$. In terms of dimensionless variables ($p=\Lambda x^{1/(D-2)},\ \delta B(p^2)=\Lambda\Sigma(x),\ g=G/\Lambda^{2-D}$), Eq.(18) is reduced to a linearized integral equation

$$
\Sigma(x)=\frac{1}{N}\int_{\sigma_m}^{1}dyK(x^{1/(D-2)},y^{1/(D-2)};g)\Sigma(y), \tag{22}
$$

where $\sigma_m=(m/\Lambda)^{D-2}$ ($0<\sigma_m\le 1$) is the rescaled infrared cutoff and in fact m is nothing but the dynamically generated mass by the normalization $m=\delta B(m^2)$. We can rigorously prove that, *if* N *is equal to the maximal eigenvalue of the kernel* (19), *then there exists a nontrivial solution* $\Sigma(x)$. Hence, for a given σ_m, each line $N(g,\sigma_m)$ on (N,g) plane depicts a line of equal dynamically-generated mass $m=\Lambda\sigma_m^{1/(D-2)}$. Therefore, the critical line is defined by $N_{\rm cr}(g)=\lim_{\sigma_m\to 0}N(g,\sigma_m)$ which separates the broken phase from the symmetric phase. It is difficult to obtain the explicit form of the critical line $N_{\rm cr}(g)$ for arbitrary g, however we can get it in the limit of infinite four-fermion coupling constant, $g\to\infty$. In this limit, the bifurcation equation (22) in (2+1)D is rewritten into a differential equation

$$
\frac{d}{dx}\Big(x^2\frac{d\Sigma(x)}{dx}\Big)=-\frac{32}{3\pi^2N}\Sigma(x), \tag{23}
$$

plus the infrared boundary condition $\Sigma'(\sigma_m)=0$ and the ultraviolet one $[\,x\Sigma'(x)+\Sigma(x)\,]_{x=1}=0$. When $N>N_{\rm cr}\equiv 128/3\pi^2$, there is no nontrivial solution of Eq.(23) satisfying the boundary conditions, while for $N<N_{\rm cr}$

the following bifurcation solutions exist:

$$\Sigma(x) = \frac{\sigma_m}{\sin(\frac{\omega}{2}\delta)} \left(\frac{x}{\sigma_m}\right)^{-\frac{1}{2}} \sin\left\{\frac{\omega}{2}\left[\ln\frac{x}{\sigma_m} + \delta\right]\right\}, \tag{24}$$

where $\omega \equiv \sqrt{N_{\rm cr}/N - 1}$, $\delta \equiv 2\omega^{-1}\arctan\omega$ and σ_m is given by the ultraviolet boundary condition:

$$\frac{\omega}{2}\left(\ln\frac{1}{\sigma_m} + 2\delta\right) = n\pi, \quad n = 1, 2, \cdots \ . \tag{25}$$

The solution with $n = 1$ is the nodeless (ground state) solution whose scaling behavior is read from Eq.(25):

$$\frac{m}{\Lambda} = e^{2\delta}\exp\left[-\frac{2\pi}{\sqrt{N_{\rm cr}/N - 1}}\right]. \tag{26}$$

The critical four-fermion number $N_{\rm cr} = 128/3\pi^2$ is the same as the one in QED_3 with the nonlocal gauge.

It is turn to comment briefly on the dynamically generated mass of the gauge boson and the dual transformation. The vector (gauge) boson is merely an auxiliary field at the tree level, however it turns out to be propagating by obtaining the kinetic term through fermion loop effect when the fermion acquires the dynamical mass. In (2+1) dimensions the pole mass M_V of the dynamical gauge boson is given by

$$\frac{1}{2\pi}\left[\frac{4m^2 + M_V^2}{2M_V}\tan^{-1}\frac{M_V}{2m} - m\right] = G^{-1}, \tag{27}$$

which always satisfies a condition that $M_V < 2m$. Furthermore, the dual gauge field H_μ in Eq.(7) shares exactly the same pole structure with the gauge field A_μ irrespectively of the phase.

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